**Assignment 2**

**Operation Research**

**BTech Sem IV (20-21 Batch)**

Q1. Describe graphically what the simplex method does step by step to solve the following problem. (4.1.6)

Maximize *Z* = 2*x*1 + 3*x*2,

subject to

-3*x*1 + *x*2 1

\_4*x*1 + 2*x*2  20

\_4*x*1 + *x*2 10

-*x*1 + 2*x*2  5

and

*x*1 0, *x*2 0.

Q2. Describe graphically what the simplex method does step by step to solve the following problem. (4.1.7)

Minimize *Z =* 5*x*1 + 7*x*2,

subject to

2*x*1 + 3*x*2, 147

3*x*1 + 4*x*2, 210

*x*1 - *x*2, 63

and

*x*1 0, *x*2 0.

Q3. Consider the following problem. (4.2.2)

Maximize *Z =* *x*1 + 2*x*2,

subject to

*x*1 + 3*x*2  8

*x*1 + *x*2  4

and

*x*1 0, *x*2 0.

**(a)** Introduce slack variables in order to write the functional constraints in augmented form.

**(b)** For each CPF solution, identify the corresponding BF solution by calculating the values of the slack variables. For each BF solution, use the values of the variables to identify the nonbasic variables and the basic variables.

**(c)** For each BF solution, demonstrate (by plugging in the solution) that, after the nonbasic variables are set equal to zero, this BF solution also is the simultaneous solution of the system of equations obtained in part (*a*).

**(d)** Repeat part (*b*) for the corner-point infeasible solutions and the corresponding basic infeasible solutions.

**(e)** Repeat part (*c*) for the basic infeasible solutions.

Q4. Consider the problem in Q3. . (4.3.3)

1. Work through the simplex method (in algebraic form) step by step to solve the model
2. Verify the optimal solution you obtained by using a software package based on the simplex method.



|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Iteration | Basic Var. |  | Equation | Z |  |  |  |  | RHS | Minimum Ratio Test | |
| 1 | Z |  | (0) | 1 | -1 | -2 | 0 | 0 | **0** |  |  |
| Solution  (0 0 8 4)  Z = 0 |  |  | (1) | 0 | 1 | **3** | 1 | 0 | **8** | **8/3** | **Min** |
| Not Optimal |  |  | (2) | 0 | 1 | 1 | 0 | 1 | **4** | **4/1** |  |
| Entering Variable : Leaving Variable: (will be set to 0) | | | | | | | | | | | |
| 2 |  |  | (0) | 1 | -1/3 | 0 | 2/3 | 0 | **16/3** |  |  |
| Solution  (0 8/3 0 4/3)  Z = 16/3 |  | 2\*Eq (1) add to Eq. (0) | (1) | 0 | 1/3 | 1 | 1/3 | 0 | **8/3** | **8** | **New Equation** |
| Not Optimal |  | Eq (1) subst from Eq. 2 | (2) | 0 | 2/3 | 0 | -1/3 | 1 | **4/3** | **2** | **Min** |
| Entering Variable : Leaving Variable: (will be set to 0) | | | | | | | | | | | |
| 3 |  |  | (0) | 1 | 0 | 0 | 1/2 | -1/2 | **6** |  |  |
| Solution  (2 2 0 0)  Z = 6 |  |  | (1) | 0 | 0 | 1 | 1/2 | 1/2 | **2** |  |  |
| Optimal |  | 1/3\*Eq (2) substract from Eq. (1), and add to Eq (0) | (2) | 0 | 1 | 0 | -1/2 | 3/2 | **2** |  | **New Equation** |
|  |  |  |  |  |  |  |  |  |  |  |  |

5. Work through the simplex method step by step (in tabular form) to solve the following problem. (**4.4-7)**

Maximize *Z* = 2*x*1 – *x*2 + *x*3,

subject to,

3*x*1 + *x*2 + *x*3 6

*x*1 - *x*2 + 2*x*3 1

*x*1 + *x*2 - *x*3 2

and

*x*1 0, *x*3 0 *x*3 0.

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Iteration | Basic Var. |  | Equation | Z |  |  |  |  |  |  | RHS | Minimum Ratio Test | |
| 1 | Z |  | (0) | 1 | -2 | 1 | -1 | 0 | 0 | 0 | 0 |  |  |
| Solution |  |  | (1) | 0 | 3 | 1 | 1 | 1 | 0 | 0 | 6 | 2 |  |
| (0 0 0 2 1 2) |  |  | (2) | 0 | 1 | -1 | 2 | 0 | 1 | 0 | 1 | 1 | Min |
| Z = 0 |  |  | (3) | 0 | 1 | 1 | -1 | 0 | 0 | 1 | 2 | 2 |  |
|  | EV |  |  |  |  |  |  |  |  | LV |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  | (0) | 1 | 0 | -1 | 3 | 0 | 2 | 0 | 2 |  |  |
| Solution |  |  | (1) | 0 | 0 | 4 | -5 | 1 | -3 | 0 | 3 | 3/4 |  |
| (1 0 0 3 0 1) |  |  | (2) | 0 | 1 | -1 | 2 | 0 | 1 | 0 | 1 | -1 | New Equation |
|  |  |  | (3) | 0 | 0 | 2 | -3 | 0 | -1 | 1 | 1 | 1/2 | Min |
|  | EV |  |  |  |  |  |  |  |  | LV |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  | (0) | 1 | 0 | 0 | 1.5 | 0 | 1.5 | 0.5 | 2.5 |  |  |
| Solution |  |  | (1) | 0 | 0 | 0 | 1 | 1 | -1 | -2 | 1 |  |  |
| (1.5 0.5 0 1 0 0) |  |  | (2) | 0 | 1 | 0 | 0.5 | 0 | 0.5 | 0.5 | 1.5 |  |  |
| Z = 2.5 |  |  | (3) | 0 | 0 | 1 | -1.5 | 0 | -0.5 | 0.5 | 0.5 |  | New Equation |

6. Consider the following problem. **(4.4-6.)**

Maximize *Z =* 3*x*1 + 5*x*2 + 6*x*3,,

subject to

2*x*1 + *x*2 + *x*3 4

*x*1 + 2*x*2 + *x*3 4

*x*1 + *x*2 + 2*x*3 4

*x*1 + *x*2 + *x*3 3

and

*x*1 0, *x*3 0 *x*3 0

**(a)** Work through the simplex method step by step in algebraic form.

**(b)** Work through the simplex method in tabular form.

**(c)** Use a computer package based on the simplex method to solve the problem.

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| Iteration | Basic Var. |  | Equation | Z |  |  |  |  |  |  | RHS | Minimum Ratio Test | |
| 1 | Z |  | (0) |  |  |  |  |  |  |  |  |  |  |
|  |  |  | (1) |  |  |  |  |  |  |  |  |  |  |
|  |  |  | (2) |  |  |  |  |  |  |  |  |  |  |
|  |  |  | (3) |  |  |  |  |  |  |  |  |  |  |
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7. Consider the following statements about linear programming and the simplex method. Label each statement as true or false, and then justify your answer. **(4.5-1)**

**(a)** In a particular iteration of the simplex method, if there is a tie for which variable should be the leaving basic variable, then the next BF solution must have at least one basic variable equal to zero.

**(b)** If there is no leaving basic variable at some iteration, then the problem has no feasible solutions.

**(c)** If at least one of the basic variables has a coefficient of zero in row 0 of the final tableau, then the problem has multiple optimal

solutions.

**(d)** If the problem has multiple optimal solutions, then the problem must have a bounded feasible region.